MPI system matrix reconstruction: making assumptions on the imaging device rather than on the tracer spatial distribution

Gael Bringout[a,*], Ksenija Gräfe[a], Thorsten M. Buzug[a]

[a] Institue of Medical Engineering, Universität zu Lübeck, Germany
[*] Corresponding author, email: bringout@imt.uni-luebeck.de

INTRODUCTION To reconstruct a first approximation of an unknown spatial tracer distribution, a preliminary solution of the inconsistent system of linear equations $\hat{S}c = \hat{u}$ (1) can be calculated, using a regularized weighted normal equation of the first kind formulated as $(\hat{S}^H WS + \lambda I)c = \hat{S}^H W\hat{u}$ (2), with $\hat{S}$ the system matrix (SM) and $\hat{S}^H$ its complex conjugate transpose, c the tracer distribution, $\hat{u}$ the acquired signal in Fourier space, W a weighting matrix, and λ the regularisation parameter together with the identity matrix I for Tikhonov regularization [1].
Furthermore, it is common to truncate the measurements according to the SNR, using a correspondingly truncated SM, $\hat{S}^\dagger$, and measurements $\hat{u}^\dagger$ in (2). However, the regularization scheme is chosen to enforce certain properties of the tracer distribution, which are in general unknown.
The presented technique does not make any assumptions on the spatial tracer distribution, besides that it can be approximated by solving directly $\hat{S}^\dagger\, c = \hat{\mathrm{u}}^\dagger$ (3). Information only coming from the scanner design, imaging sequence, and acquired signal is used to truncate the matrices. The truncation thresholds can then be used for several measurements.

MATERIAL AND METHODS A single-sided scanner encoding a 2D plane by moving a low field volume along a 2D Lissajous curve is used [2, 3]. The measurements are used in form of the power spectrum of the acquired signal. The SM is measured on $M = 225$ positions $\vec{r}$ equidistantly spaced on a 2D grid.
In a first truncation step, data are rejected according to an SNR measurement defined for each frequency component k as

$$\mathrm{SNR}(k) = \frac{\|\hat{u}_k\|}{std(Empty_k)},$$

with $std(Empty_k)$ the standard deviation determined from several air measurements. Here, a hard threshold of SNR > 10 is applied on the data coming from each receive channel.
Then, the energy $\widetilde{w}_k$, defined by

$$\widetilde{w}_k = \sum_{p=1}^{M} \left\|\hat{S}_\mathrm{k}(\vec{r}_p)\right\|^2,$$

is used to further truncate the measurements, retaining the frequency components with the highest energy [4]. A hard threshold of $\widetilde{w}_k > 0.01$ is applied on each channel. Arbitrary units are used due to the missing calibration of the receive channels.
Finally, the orthogonality map [1, 4] or Gramian matrix [5], calculated between the frequency components i and j of the SM as

$$G_{ij} = \langle \hat{s}_i, \hat{s}_j \rangle = \left\| \sum_{p=1}^{M} \frac{\hat{s}_i(\vec{r}_p)}{\|\hat{s}_i\|} \frac{conj(\hat{s}_j(\vec{r}_p))}{\|\hat{s}_j\|} \right\|,$$

is evaluated and used to derive another two hard thresholds. They are based on the standard deviation and the mean value of the Gramian matrix for a given i and any j. They have to be smaller than 0.06 and 0.07, respectively.
A modified Kaczmarz method is used to solve (3), imposing a real and non-negativity constraint on the solution, which leads to an approximation c^p of c after p iterations.

RESULTS Fig. 1 compares the reconstruction of two phantoms (Fig. 1, left) solving either equations (2) (Fig. 1, middle) or (3) (Fig. 1, right). The latter distributions are obtained after a calculation time of 164 ms and 143 ms on an Intel i5-760. The reconstructed distributions look similar or better for the presented approach. See imt.uni-luebeck.de or [6] for the corresponding scripts and data.

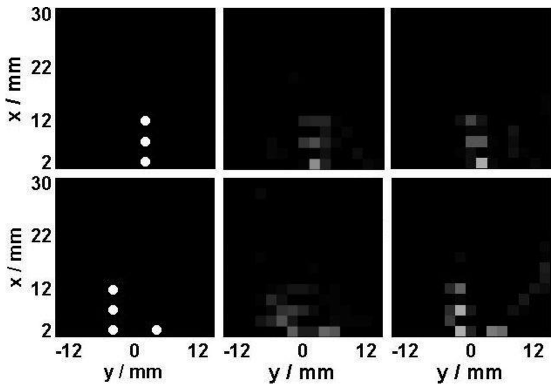

Figure 1: From left to right: model of the used phantoms. Reconstructions with a weighted regularized least squares approach with $\lambda \approx 4.65 \cdot 10^{-6}$ and $\lambda \approx 5.86 \cdot 10^{-6}$ and stopped after 5 iterations. Reconstruction with the presented method stopped after 50 iterations. All images use the same colour scale.

CONCLUSION The presented reconstruction technique solved the inconsistent system only by truncating the acquired data and by early stopping the iterative solver. Doing so, no assumption on the tracer distribution is made. The used thresholds can all be related to quantities, which can be linked to the scanner properties. Moreover, this technique does not seem to reduce the quality of the reconstruction. This strategy shows a way to the development of automatic techniques designed to obtain first and reliable reconstructions of the tracer distribution.

ACKNOWLEDGEMENTS The authors thank Christina Brandt and Hanne Medimagh for the fruitful discussion and gratefully acknowledge the financial support of the German Federal Ministry of Education and Research (BMBF, grant numbers 13N11090 and 01EZ0912) and of the European Union and the State Schleswig-Holstein (Programme for the Future - Economy, grant number 122-10-004).

REFERENCES

[1] T. Knopp and T. M. Buzug, Springer-Verlag, Berlin Heidelberg, 2012. ISBN 978-3-642-04199-0.
[2] T. F. Sattel et al., *Journal of Physics D: Applied Physics*, 42(2):1-5, 2009. doi: 10.1088/0022-3727/42/2/02200110.1109.
[3] K. Gräfe et al., 2015 5th International Workshop on Magnetic Particle Imaging (IWMPI). doi: 10.1109/IWMPI.2015.7107024.
[4] G. Bringout, Universität zu Lübeck, 2015, submitted. Ph.D. Thesis.
[5] N. Traulsen, Infinite Science Publishing, Lübeck. 2015.
[6] Data available under github.com/KsenijaGraefe/SingleSidedData.