Coil Design for Magnetic Particle Imaging: Application for a Pre-clinical Scanner

Gael Bringout, Member, IEEE, and Thorsten M. Buzug, Member, IEEE

A method to design electromagnetic coils for magnetic particle imaging (MPI) scanners is presented. Three different coils for a rabbit sized scanner are designed, namely a cylindrical drive coil, a biplanar drive coil, and a quadrupole. Based on the inverse boundary elements method, the optimization of all coils used in an MPI scanner is possible. The inverse problem is solved using the Tikhonov regularization and a quadratic programming with quadratic constraints algorithm. Depending on the chosen objectives, the coils can be designed to precisely generate a given field, having a small inductance, resistance, or having a smooth winding pattern. The properties of a cylindrical drive coil are then validated on a prototype. The use of this technique allows the optimization of the scanner design, to enhance the imaging capacity, and to easily gain control over the key properties of the coils in order to scale up scanners to human size.

Index Terms—MPI, coil design, inverse boundary elements, drive coil, quadrupole, quadratic programming, Tikhonov regularization.

I. INTRODUCTION

Magnetic particle imaging (MPI) is a new imaging technique to visualize a volume containing a distribution of magnetic nanoparticles [1]. The first in vivo experiments performed with mice have demonstrated that this technique is able to acquire data with high sensitivity, high resolution, and high speed [2].

In order to detect the nanoparticles in a volume, a homogeneous sinusoidal magnetic field, the drive field, is generated as illustrated in Fig. 1. Due to the non-linear magnetization of the nanoparticles, the received signal is altered, i.e. the signal spectrum contains harmonics, with the energy of the harmonics being proportional to the nanoparticles concentration.

To spatially encode the signal, a magnetic gradient field, the selection field, is superimposed to the drive field. This field contains a field free space (FFS). In close vicinity of the FFS, the nanoparticle magnetization is affected by the drive field and harmonics are created. Particles further away from the FFS are saturated and do not create any higher harmonics (see Fig. 2). In order to encode a volume, the FFS is moved in space by varying the amplitude of the drive field.

Two types of FFS are used. It may be either a field free point (FFP) or a field free line (FFL). In an FFP scanner, the selection field is constant in time and drive fields are used with different frequencies to move the FFP along trajectories such as a Lissajous or a Cartesian curve [3], [4]. In an FFL scanner, the selection field is rotated at a much lower frequency than the drive fields. This rotation allows the use of an acquisition scheme similar to the one used in a computed tomography scanner [5]–[8]. Up to now, different implementation concepts exist. An overview of these concepts is presented in [9].

The drive coil generates the drive field, which varies sinusoidally in time with a single frequency. This frequency is fixed for each scanner, ranging from 1 kHz to 150 kHz and has an amplitude of a few tenth of millitesla to a few millitesla. The selection coil, which generates the selection field, is constant in time and drive fields are used with different frequencies to move the FFP along trajectories such as a Lissajous or a Cartesian curve [3], [4]. In an FFL scanner, the selection field is rotated at a much lower frequency than the drive fields. This rotation allows the use of an acquisition scheme similar to the one used in a computed tomography scanner [5]–[8]. Up to now, different implementation concepts exist. An overview of these concepts is presented in [9].

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mainly a solenoid or a dipole in the case of FFP scanners and quadrupoles in case of FFL scanners. The gradient amplitude varies from a few hundreds of millitesla per meter to several tesla per meter. The selection field is either static or may have a low rotation speed in case of FFL scanners.

Moreover, the field amplitude tends to be increased. Indeed, the higher the field amplitudes, the higher is the maximal resolution and the field of view [10], [11]. But, different challenges remain. The upscaling of the coils toward human sized MPI scanners and the use of magnetic fields with high frequencies and/or high amplitudes encourage the design of optimized coils. Such an optimization has multiple objectives. It may either keep the dissipated power at a reasonable level to simplify the cooling of the system, increase the thermal stability of the scanner, limit the currents amplitude to reduce the constraints on the other elements of the system, lower the voltage amplitude, or simplify the scanner construction. Besides, in an FFL scanner the existence and quality of the line is strongly related to the fields properties. The linearity and homogeneity of those fields has thus to be considered during any coil optimization for FFL scanners.

To date, different methods exist to model, parametrize, and solve such optimization problems [12]–[18]. We have decided to use a technique based on the boundary element method (BEM) [14]–[18], which uses a meshed surface as support, thus allowing any type of form as support. The same design technique can be used to optimize any type of coil needed in an MPI scanner and is more versatile as other design techniques currently used for MPI scanner [19], [20]. In order to fulfill the quasi-static hypothesis used by those techniques, the coils were designed to use litz wire when needed.

In this paper, three coils for a rabbit sized MPI scanner are designed, namely a cylindrical drive coil, a cylindrical quadrupole for an FFL scanner, and a planar drive coil for an FFP scanner. The simulated properties for the cylindrical drive coil are validated based on measurements with a prototype.

II. Methods

The aim of the coil design technique presented here, is to find the most suitable coil, which generates a desired magnetic flux density $B$ in a volume $V_{\text{target}}$.

This is done in three steps. First, the optimal current density $J$ is obtained through a single optimization problem, which can be regularized using a Tikhonov regularization [18], [21] scheme and solved using a linear or quadratic programming algorithm [22]. Once an optimal $J$ is found, the wire loops are approximated and a wire path is created. This step takes other constraints into consideration, as the wire width, flexibility, the cooling methods, the specification of other parts as filter or the coil construction techniques. If some construction constrains cannot be met, the optimization problem have to be adapted. If all the construction constrains and minimal coil performances are met, the most suited coil can be build and validated.

In this section, all steps from the definition of a given geometry and magnetic field to the coil validation will be explained in detail. In subsections II-A to II-E the problem is described using continuous functions describing a current density. In II-F to II-I the problem is discretized and solved. Then from II-J to II-K, the solution is reworked to obtain the coil centroids and important physical properties are calculated. Finally, the coils can be built and their properties validated.

A. Forward problem

A conducting volume $V_{\text{cond}}$ with a conductivity $\rho$ and a thickness $d$, supporting $J$, produces in a different volume with an electric permittivity $\epsilon$ and a magnetic permeability $\mu$, a magnetic flux density $B$ and an electrical field $E$ according to the Maxwell relation [23]

$$\text{curl} \left( \frac{B}{\mu} \right) = J + \frac{\partial (\epsilon E)}{\partial t},$$

(1)

The design technique is done under the quasi static hypotheses. Thus the time dependent part of (1) can be ignored. $J$ is chosen constant in time. This leads to

$$\text{curl} \left( \frac{B}{\mu} \right) = J,$$

(2)

which can be reformulated as the Biot-Savart law [23]

$$B(r_0) = \frac{\mu}{4\pi} \int_{\mathbb{R}^3} \frac{r - r_0}{|r - r_0|^3} \times J(r) dV;$$

(3)

with $r = [x \ y \ z]^T \in V_{\text{cond}}$ and $r_0 = [x_0 \ y_0 \ z_0]^T$ in the target field volume. The magnetic field is given as a function of the current density. By inversion of (3), $J$ will be given as a function of $B$.

B. Representation of the current density as series

$J$ has to be reformulated to facilitate its later expression on a mesh. It can be expressed as the series [22]

$$J(r) = \sum_{n=1}^{\infty} \hat{J}_n(r),$$

(4)

with $(\hat{J}_n(r))_{n \in \mathbb{N}}$ being a suitable set of independent basis functions, which guarantees that

$$\text{div} \ \hat{J}_n = 0.$$  

(5)

This means that $J$ is source-free, i.e. it represents a physically plausible solution, where no charges are created nor destroyed.

C. Objective and constraint functions

In order to optimize the current density, objective and constraint functions have to be evaluated. In this work, four quantities are used for this purpose. Two are quadratic functions of $J$, namely the dissipated power $P_{\text{dis}}$ and the stored energy $E_{\text{stored}}$. The two others are linear functions of $J$, namely the Laplacian of the current density, $\nabla^2 J$, and the magnetic field amplitude, $B$. The latter is already presented in (3), the three others are described in detail here.
1) Stored energy - mutual inductance

An operator able to evaluate the mutual inductance between the elements of \( \mathbf{J} \) is defined based on the stored energy in \( V_{\text{cond}} \). The stored energy \( E_{\text{stored}} \) of a current \( I \) through a coil with an inductance \( L \) is given by

\[
E_{\text{stored}} = \frac{1}{2} LI^2. \tag{6}
\]

The inductance of \( V_{\text{cond}} \) is given by the mutual inductance between the basis functions \( \mathbf{J}_m \) and \( \mathbf{J}_n \). It can be expressed as [14]

\[
M_{mn} = \frac{\mu_0}{4\pi} \int_{V_m} \int_{V_n} \mathbf{J}_m(r) \cdot \mathbf{J}_n(r') \frac{dV'}{dV}. \tag{7}
\]

Thus, minimizing the stored energy in the coil will also minimize the inductance.

2) Dissipated power - mutual resistance

Similar to the inductance, the resistance is estimated from the dissipated power \( P_{\text{dis}} \) as

\[
P_{\text{dis}} = RI^2, \tag{8}
\]

with \( R \) being the resistance of the coil. The mutual resistance between the basis function \( \mathbf{J}_m \) and \( \mathbf{J}_n \) is given by

\[
R_{mn} = \rho \int_{V_m} \int_{V_m} \| \mathbf{r}_m - \mathbf{r}_n' \| \frac{dV'}{dV}, \tag{9}
\]

if the two basis functions \( \mathbf{J}_m \) and \( \mathbf{J}_n \) share a common element [22], [24]. Otherwise, the mutual resistance is zero. Thus, minimizing the dissipated power will also minimize the resistance of the coil.

3) Laplacian

The Laplacian of the current density is a scalar field defined for every point on a surface, that evaluates the smoothness of the current density. Minimizing the Laplacian will reduce the geometrical variation of the surface current density. This favors straight wire paths over curved ones, hence facilitating the fabrication of the coil. The Laplacian of the current density is expressed as

\[
\nabla^2 \mathbf{J} = \nabla^2 \mathbf{J} (\mathbf{r}). \tag{10}
\]

D. From volume to surface

To simplify the representation of the conducting volume, the thickness \( d \) is assumed to be small compared to the other dimensions. Thus, the conducting volume \( V_{\text{cond}} \) can be represented by a surface \( S \). \( \mathbf{J} \) is then replaced by the surface current density \( \mathbf{j} \)

\[
\mathbf{j}(\mathbf{r}) = d(\mathbf{r}) \mathbf{J}(\mathbf{r}), \quad \text{with } \mathbf{r} \in S. \tag{11}
\]

E. Stream function

To accurately calculate the conductor path on the surface, \( \mathbf{j} \) should be expressed in terms of a stream function \( \psi \) [14], [15]. It follows that

\[
\mathbf{j}(\mathbf{r}) = \text{curl}(\psi(\mathbf{r}) \mathbf{n}(\mathbf{r})), \tag{12}
\]

with \( \mathbf{n}(\mathbf{r}) \) being the normal of the surface \( S \) at position \( \mathbf{r} \).

F. Discretization

In order to discretize the problem on a mesh with \( N \) nodes, a finite set of basis stream functions \( \{\hat{\psi}_n(\mathbf{r})\}_{n=1}^N \), which are piecewise continuously differentiable on \( S \) [22], is introduced. \( \psi \) is represented by a set of basis functions,

\[
\psi(\mathbf{r}) = \sum_{n=1}^N s_n \hat{\psi}_n(\mathbf{r}), \tag{13}
\]

with \( s_n \) being the stream function amplitude at position \( \mathbf{r} \). Considering \( N \) nodes with coordinates \( \mathbf{r}_n, n = 1, \ldots, N \) on \( S \), the function \( \hat{\psi}_n \) at node \( n \) is defined as

\[
\hat{\psi}_n(\mathbf{r}_j) = \begin{cases} 1 & \text{if } j = n, \\ 0 & \text{if } j \neq n, \end{cases}, \quad j, n = 1, \ldots, N. \tag{14}
\]

and \( \hat{\psi}_n \) decreases linearly on the edges connected to the node \( n \). The vector \( \mathbf{s} = [s_1 \ s_2 \ \ldots \ s_N]^T \) is called the stream function vector.

In this work, \( S \) will be formed by \( K \) triangular elements and obtained via the modelization software Blender (Blender 2.68a, Blender Foundation, Netherlands).

The surface current density on the triangle \( k \) connected to the nodes \( \{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3\} \) with the basis \( \{\hat{\psi}_1, \hat{\psi}_2, \hat{\psi}_3\} \) is given by [22]

\[
\mathbf{j}_k = \frac{s_1 \hat{\psi}_1(\mathbf{r}_3 - \mathbf{r}_2) + s_2 \hat{\psi}_2(\mathbf{r}_1 - \mathbf{r}_3) + s_3 \hat{\psi}_3(\mathbf{r}_2 - \mathbf{r}_1)}{\| (\mathbf{r}_2 - \mathbf{r}_1) \times (\mathbf{r}_3 - \mathbf{r}_1) \|.} \tag{15}
\]

The surface current density can thus be expressed by

\[
\mathbf{j}(\mathbf{r}) = \sum_{k=1}^K \mathbf{j}_k, \tag{16}
\]

which permits the reformulation of the objective and constraint functions as matrices in the set of basis functions, \( \{\hat{\psi}_n(\mathbf{r})\}_{n=1}^N \), on a mesh.

G. Matrix formulation

The objective and constraint functions needed to optimize the coil have to be expressed on the mesh. Various challenges exist for their calculation. A review and possible solutions is given in [24].

1) Matrix \( \mathbf{M} \)

The mutual inductance can be expressed by

\[
E_{\text{stored}} = \frac{1}{2} \sum_{m=1}^N \sum_{n=1}^N M_{mn} s_n s_m. \tag{17}
\]

The elements \( M_{mn} \) are assembled into a positive definite matrix \( \mathbf{M} \in \mathbb{R}^{N\times N} \), which is used to rewrite (17) as

\[
E_{\text{stored}} = \frac{1}{2} \mathbf{s}^T \mathbf{M} \mathbf{s}. \tag{18}
\]

2) Matrix \( \mathbf{R} \)

The mutual resistance can be expresses as

\[
P_{\text{dis}} = \sum_{m=1}^N \sum_{n=1}^N R_{mn} s_n s_m. \tag{19}
\]

The elements \( R_{mn} \) are assembled into the positive definite matrix \( \mathbf{R} \in \mathbb{R}^{N\times N} \), which is used to rewrite (19) as

\[
P_{\text{dis}} = \mathbf{s}^T \mathbf{R} \mathbf{s}. \tag{20}
\]
3) Matrix C
The relation (3) can be split into its three components, giving one matrix per component.

\[
B_x(r_0) = \frac{\mu}{4\pi} \int \frac{(z - z_0)j_y(r) - (y - y_0)j_z(r)}{||r - r_0||^3} dS
= \sum_{n=1}^{N} C_{x_n}(r_0) s_n, \tag{21}
\]

\[
B_y(r_0) = \frac{\mu}{4\pi} \int \frac{(x - x_0)j_z(r) - (z - z_0)j_x(r)}{||r - r_0||^3} dS
= \sum_{n=1}^{N} C_{y_n}(r_0) s_n, \tag{22}
\]

\[
B_z(r_0) = \frac{\mu}{4\pi} \int \frac{(y - y_0)j_x(r) - (x - x_0)j_y(r)}{||r - r_0||^3} dS
= \sum_{n=1}^{N} C_{z_n}(r_0) s_n, \tag{23}
\]

The matrices \( C_x, C_y, \) and \( C_z \) are then assembled from the values \( C_{x_n}, C_{y_n}, \) and \( C_{z_n} \), respectively, \( C_x, C_y, \) and \( C_z \in \mathbb{R}^{T \times N} \), with \( T \) being the number of target points on which the magnetic field amplitude is calculated. Relations (21)-(23) can be expressed in matrix form as

\[
B = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} s. \tag{24}
\]

4) Matrix \( \mathcal{L} \)
The Laplacian of the surface current density at the point \( m \) can be expressed by

\[
\mathcal{L}j(r_m) = \frac{1}{4\pi h^2} \sum_{k=1}^{K} \frac{\text{Area}(k)}{3} \sum_{n=1}^{N} e^{-\frac{||r_n - r_m||}{\eta}} (j(r_n) - j(r_m)), \tag{25}
\]

with \( h \) being a positive quantity defining the size of the neighborhood [25].

The elements \( \mathcal{L}j \) can be assembled into the matrix \( \mathbf{L}_S \in \mathbb{R}^{N \times N} \) to form the system of linear equations

\[
\mathbf{L} = \mathbf{L}_S s. \tag{26}
\]

5) Target field
The field, which has to be produced by the surface current density, is called the target field \( \mathbf{B}_{\text{target}} \) and is given by the relation

\[
\mathbf{B}_{\text{target}} = \begin{pmatrix} B_{x_{\text{target}}} \\ B_{y_{\text{target}}} \\ B_{z_{\text{target}}} \end{pmatrix}, \tag{27}
\]

with \( B_{x_{\text{target}}}, B_{y_{\text{target}}}, B_{z_{\text{target}}} \in \mathbb{R}^{T} \) being the \( x, \) \( y, \) and \( z \) components of the target field.

All functions of the forward problem are now expressed as matrices. But, the boundary conditions have still to be integrated, in order to accurately represent the functions.

H. Boundary condition
The boundary condition applies on all the boundaries of the surface. In order to guarantee that \( \mathbf{j} \) is divergence free, the stream function \( \psi \) has to be constant and equal to zero on each closed sub-boundaries of \( S \) [22].

The forward problem is now completely expressed on the mesh, can be inversed, and solved.

I. Solving the inverse problem
In order to solve the inverse problem, two different methods are used. For the sole minimization of the field error, the ill-posed problem is solved using a Tikhonov regularization scheme. For minimization problems involving quadratic relation of \( s \), as the stored energy or the dissipated power, a quadratic programming scheme is used.

1) Tikhonov regularization
Considering relations (24) and (27), the Tikhonov regularization leads to the minimization problem [21]

\[
\min_s \left\{ \| \mathbf{B} - \mathbf{B}_{\text{target}} \|^2 + \lambda^2 \| \mathbf{T} s \|^2 \right\}, \tag{28}
\]

with the matrix \( \mathbf{T} \in \mathbb{R}^{N \times N} \) being the Tikhonov matrix. The Tikhonov matrix gives a preference to solutions based on the norm of the linear relation \( \mathbf{T} s \). When using the matrix \( \mathbf{L}_S \) as the Tikhonov matrix, the solution will be smoother. When using the identity matrix as the Tikhonov matrix, the solution with a small norm will be preferred. The factor \( \lambda \) is called the regularization parameter and is chosen experimentally.

This means that the surface current density to be built is not necessarily the true solution of the minimization problem. A specialized Matlab (Matlab R2010b, MathWorks, USA) toolbox [26], [27] has been used to solve the minimization problem (28).

2) Quadratic programming
The optimization problem can also be formulated as a quadratic programming problem with quadratic constrains (QPQC). This will be, for example, the minimization of the stored energy, while keeping \( \mathbf{B} \) into a 10% error margin compared to \( \mathbf{B}_{\text{target}} \) and having dissipated power below 1000 W. This can be written as

\[
\min_s \frac{1}{2} s^T \mathbf{M} s
\]

subject to: \( \mathbf{B}_{\text{Tinf}} \leq \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} s \leq \mathbf{B}_{\text{Tsup}} \)

\[
s^T \mathbf{R} s \leq 1000 \tag{29}
\]

and solved using the Matlab toolbox OPTI TOOLBOX [28] and the solver IPOPT [29].

J. Wire approximation
To approximate \( \mathbf{j} \) as one single wire, a set of wire loops \( s_n \) has to be extracted first. According to the number of sets of loops \( N_{\text{loops}} \in \mathbb{N}^+ \), the current flowing in a loop can be defined with the relation

\[
I_c = \frac{\max(s) - \min(s)}{N_{\text{loops}}}. \tag{30}
\]
The centerline of each set of loops \( s_n \) is then given by

\[
s_n = \min(s) + (n - \frac{1}{2})L_b, \quad n = 1, \ldots, N_{\text{loop}}.
\]

Once the centerlines are obtained, Solidworks (SW 2010, Dassault systèmes, France) is used to manually assemble them in a single wire path. The results of this process can be seen in Fig. 3.

![Wire discretization process from the solution on a cylinder to the wire path. (a) Top view of a cylinder with a surface current density optimized with a Tikhonov regularization using the identity matrix to produce a drive coil (i.e. a homogeneous \( B_0 \) fields). The white color corresponds to a surface current density of zero and the black color of the maximal surface current density. (b) Planar projection of the wire loops centerlines. (c) Planar projection of the wire path approximation.](image)

K. Coil properties evaluation

Calculations have been done in Matlab (Matlab 7.11.0 64bit, Mathworks, Natick, USA). The implementation is available on http://www.imt.uni-luebeck.de and http://github.com/gBringout/CoilDesign. Once the wire loops or the wire paths are obtained, the inductance is evaluated using FastHenry (FastHenry 3.32, FastFieldSolvers S.R.L, Italy) [30]. The length of the path is then calculated as the sum of each coil segments length. The resistance is obtained by multiplying the length by the resistance per meter of the used conductor. The field is calculated from the centerlines using a Biot-Savart implementation in Matlab.

L. Quasi-static work-around / Litz wire

Due to the fact that a signal in the kilohertz range is used for the drive coil, the quasi static hypotheses does not hold for such coils. The main effect is the increased impact of the skin effect on the coil resistance [23]. In order to limit this increase, the coil is wound with a litz wire specially designed for this application. A typical litz wire used in MPI is made of many thousands of single isolated wires with a copper diameter of a few tenth of micrometer. For a given wire cross section, the use of litz wire limits the resistance increase as a function of the frequency. This minimize the power dissipated at the sending frequency and receiving frequencies. So, the litz wire design is a trade-off between the single wire diameter, production cost and high-frequencies (>1 kHz) resistance. Due to the insulation material of each single wire, the DC resistance of a litz wire is increased by a factor of approximatively two when compared to the one of a copper wire of equivalent cross section.

M. Coil properties validation

In order to validate the numerical simulation, a prototype of the cylindrical drive coil is build. The wire path obtained in 3c are scaled to the inner diameter of the coil, printed and manually engraved on a PVC cylinder. Then, the litz wire is wound and glued on the engraved path. The inductance and resistance are measured with an high precision LCR meter (E4980A, Agilent Technologies, USA). The resonance frequency is measured with an impedance analyser (4194A, Hewlett-Packard Company, USA). A DC Current source (SM800 7.5-80, Delta Elektronika, Netherlands) is used to supply an appropriate current for the measurements. The current amplitude is continuously measured with a current clamp (A622, Tektronix, USA) connected to an oscilloscope (DPO 3034, Tektronix, USA). A current of 10.0 A was used to produce a magnetic field amplitude high enough to enable proper measurements and to keep the dissipated power low. The magnetic field was measured with a Hall probe and a Gaussmeter (MMZ-2508-UH and Model 460, Lake shore, USA) using a robot (3D set of linear axis, isel Germany AG, Germany).

III. Results

Three coil sets have been designed using the coil design techniques proposed in this contribution, one of which have been build. Each of the coil sets is an example for a given MPI scanner geometry, with different requirements. The cylindrical drive coil should have a length smaller than 0.4 m and a field homogeneity in a sphere of 0.08 m of about 10 %. The biplanar drive coil should have an efficiency as high as possible, for a length of 0.25 m. The requirements are summarized in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>MODEL USED FOR THE DESIGN.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimization goal</td>
<td>Cylindrical drive coil</td>
</tr>
<tr>
<td>Diameter (m)</td>
<td>0.2</td>
</tr>
<tr>
<td>Disk distance (m)</td>
<td>0.317</td>
</tr>
<tr>
<td>Target field amplitude</td>
<td>15 mT</td>
</tr>
<tr>
<td>Target field direction</td>
<td>z</td>
</tr>
<tr>
<td>Target diameter (m)</td>
<td>0.08</td>
</tr>
<tr>
<td># node</td>
<td>2501</td>
</tr>
<tr>
<td># triangle</td>
<td>4880</td>
</tr>
<tr>
<td># target point</td>
<td>515</td>
</tr>
</tbody>
</table>
**A. Cylindrical drive coil**

The Tikhonov regularization (28) using $\Gamma = I$ with $\lambda = 4.0 \times 10^{-6}$ has been chosen to solve this problem as it delivers coils with a small length even with an unconstrained mesh. The choice of the regularization parameter has to be experimentally determined, as the L-curve is ill-shaped. In Table II, the different properties of $s$ and of the discretized coil shown in Fig. 4 are given. As this is a drive coil used with a signal frequency of 25 kHz, it is made of litz wire, which explains the high DC resistance as shown in Table II. The litz wire has a width of 7.8 mm, the coil is thus made of 18 loops, which are spaced by a minimum of 8.5 mm. The obtained magnetic field $B_y$ in Fig. 4b shows a 10 % homogeneity in a sphere of roughly 8 cm in diameter. Moreover, this coil generates a drive field amplitude of 15 mT peak, with a peak current of 295 A, a peak voltage of 1070 V, and an RMS dissipated power of 542 W.

To validate those properties, a prototype, shown in Fig. 4c was built. A 7.8x7.8 mm² square litz wire made of 10000 single filaments of 63 μm in diameter was produced for this purpose (custom product, Sofilec, France). The coil has an efficiency of 53.0 μT/A, a resistance of 10.1 mΩ at DC and of 13.3 mΩ at 25 kHz and 22°C, an inductance of 30.7 μH and a self-resonance frequency of 11 MHz. The magnetic field $B_y$ was measured on a grid of 41x41 elements covering a square of 60x60 mm² with a resolution of 3 mm. Due to the difficulties to align the robot and the coil coordinate system, the measured area is not perfectly centered. The extent of the measured surface was limited by the robot arm size and the thickness of the PVC former. To ensure a good stability, the robot movement was stopped for 5 seconds at each measurement. The whole measurements took around 200 minutes. The results are shown in Fig. 4d.

**B. Biplanar drive coil**

The Tikhonov regularization (28) using $\Gamma = L$ with $\lambda = 4.0 \times 10^{-8}$ has been chosen to solve this problem as the resulting coils had a simpler wire pattern. The choice of the regularization parameter has to be experimentally determined, as the L-curve is ill-shaped. In Table II, the different properties of $s$ and of the discretized coil shown in Fig. 5 are given. As this is also a drive coil used with a signal frequency of 25 kHz, it is made of litz wire. The foreseen litz wire has a width of 3.9 mm, the coil should be made of 16 loops, spaced by a minimum of 4.9 mm. The simulated magnetic field $B_y$ in Fig. 5b shows a 10 % homogeneity in a sphere of roughly 10 cm in diameter. In order to generate a field amplitude of 3 mT, a peak current amplitude of 570 A is required.

**C. Cylindrical quadrupole**

In order to obtain a coil with a high efficiency, the minimization of the stored energy according to (29) has been chosen as minimization goal. To obtain a quadrupole topology, the matrices $C_x$ and $C_y$ model a gradient field with an amplitude of 0.5 T/m in the $x$ and $y$ direction, respectively. The matrix $C_z$ models a field with no amplitude. This describes a quadrupole field, with a field free line in $z$ direction. The field topology is thus different from the one of a Maxwell-gradient coil set. Moreover, a field error of 10 % and a dissipated power smaller than 10 kW are required. This results in a smoother $s$, on which more loops were placed. Thus the efficiency was increased in comparison to other minimization goals. However, it is expected that the actual dissipated power of the coil will be higher, as a discrete wire will be used to wound the coil. In Table II, the different properties of $s$ and of the discretized coil shown in Fig. 6 are given. As this is an FFL selection...
field coil used with a signal frequency of about 100 Hz, it can be made with an hollow copper conductor. The increased resistance due to the skin effect is negligible and the heat will be more easily extracted. The chosen conductor is square with a width of 6 mm, the coil is made out of 44 loops, each spaced by a minimum of 6.3 mm. The sum of the gradients of the magnetic field absolute values illustrated in Fig. 6b shows a 5 % linearity in a sphere of roughly 5 cm in diameter. To generate a gradient amplitude of 0.5 T/m, a current amplitude of 1302 A and a voltage of 70 V are needed, with the RMS dissipated power reaching 16 kW.

![Fig. 6. Cylindrical quadrupole generating a field free line in the z direction made of 4 segments. The arrows show the current flow direction. (a) Top view of the wire loop centerlines. (b) Side view of the wire loop centerlines and the gradient of the absolute value of the magnetic flux density in the xy plane at z=0. The dashed lines define the volume in which the targeted points were defined. The plain lines show, starting from the middle of the figure, the 5, 10, and 20 % linearity lines. The current direction is inverted in the top and bottom part compared with the one in the left and right part.](image)

### TABLE II

**Properties of the surface current density (on the top) and of the wire loops (on the bottom). Value in parentheses are measured one.**

<table>
<thead>
<tr>
<th>Optimization</th>
<th>Drive coil cylindrical</th>
<th>Drive coil biplanar</th>
<th>Quadrupole</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{dis}}$ (W)</td>
<td>Tikhonov</td>
<td>284</td>
<td>47</td>
</tr>
<tr>
<td>$E_{\text{stored}}$ (J)</td>
<td>1.5</td>
<td>0.05</td>
<td>60.5</td>
</tr>
<tr>
<td>$I_{c}$ (A)</td>
<td>291.9</td>
<td>87.5</td>
<td>998.7</td>
</tr>
<tr>
<td>Nominal field</td>
<td>15 mT</td>
<td>3 mT</td>
<td>0.5 T/m</td>
</tr>
<tr>
<td>Efficiency</td>
<td>51.2 (55.0) μT/A</td>
<td>5.2 μT/A</td>
<td>0.4 mT/(m²-A)</td>
</tr>
<tr>
<td>Current (A)</td>
<td>295</td>
<td>570</td>
<td>1302</td>
</tr>
<tr>
<td>Wire length (m)</td>
<td>10.5</td>
<td>13.4</td>
<td>23.3</td>
</tr>
<tr>
<td>$R_{\text{RC}}$ (mΩ)</td>
<td>8.8 (10.1)</td>
<td>22.4</td>
<td>13.3</td>
</tr>
<tr>
<td>$L$ (μH)</td>
<td>23.1 (30.1)</td>
<td>18.1</td>
<td>89.3</td>
</tr>
<tr>
<td>$P_{\text{RMS}}$ (W)</td>
<td>542 (753)</td>
<td>5146</td>
<td>15942</td>
</tr>
</tbody>
</table>

### IV. DISCUSSION

The numerical optimization of three different coils for MPI scanners has been carried out. The validation of the numerical calculation has been done on a cylindrical drive coil. The AC, DC resistance and inductance differences between the wire loops obtained and the actual coil are of 51 %, 15 %, and 30 %, respectively. The higher resistances and inductance comes from the imperfection of the litz wire and the extra length of wire used to wound the coil. Those have been kept during the construction in order to integrate the coil in a scanner. The 3 % efficiency increase of the build coil may be associated to the actual thickness of the coil. Also, the 750 W RMS of dissipated power and 1 kV peak voltage of this coil present no technical difficulties and can be easily integrated into a scanner. The 11 MHz self-resonance is above the expected tracer signal bandwidth, which is expected to go from 0 to 2 MHz. The homogeneity profile shows satisfactory agreement with the simulated one, considering the way the connection between the loops have been done. However, as the precise requirement in terms of homogeneity and linearity for the FFL scanner are yet undetermined, it is hard to say more about the homogeneity profile. But, as the presented technique permits the design of coils with controlled field properties, an impact study of the field topology on the image quality will be easier to realize.

In the case of an upscaled system, the integration of biplanar drive coil seems to be too challenging, as the efficiency is much smaller as the one of cylindrical coils and the obtained wire path still presents sharp variation which are unlikely to be build.

Regarding the quadrupole, the use of a hollow conductor allows the dissipation of the 16 kW. But in the case of an MPI scanner, the other components have also to be taken into account. Particularly, the filters design is affected by the current amplitude. The required 1302 A being too demanding, the coil will have to be split into two or three layers in order to reduce the current amplitude, which will reduce the dissipated power. Taking into account the connection of each segment, a 2 layers design is more suitable for the coil fabrication. Indeed, the connection of each dual-layer segment are then placed on one side, enabling the connection of each double-layer segment in order to reflect the current flow direction shown in Figure 6a.

Additionally, it has to be noted that the obtained cylindrical drive coil can easily be used to produce a high sensitivity receive coil by simply increasing $N_{\text{loops}}$ and using a smaller wire during the fabrication. In a similar way, a given surface current density can be discretized into $N_{\text{loops}}$, to achieve a given inductance, resistance or wire length. This is of main importance, especially for the cooling of the coils. Indeed, the thermal stability of the coils has to be guaranteed during the measurement process, to ensure repeatable results. This thermal stability can be influenced by varying the amplitude of the current and the length of the cooling path.

Also, we are able to produce coils with small resistance and/or impedance or even with a given quality factors. This is particularly important for the upsampling of the scanner, as the voltage and current amplitude will sharply increase compared to the rabbit sized coil presented in this contribution.

More generally, two main approaches have been used to solve the inverse problem of coil design based on a target field. The Tikhonov approach seems to be less versatile as the QPQC approach. Indeed, as the minimization is done on the field error, this method will tend to deliver coils, which produce fields similar to the ideal one, but often infeasible to construct. The QPQC put the incentive of the minimization on properties, which are more important for MPI scanners than the field quality.
V. Conclusion

In this paper, the inverse BEM method has been used to design different coils needed in MPI scanners and results were validated through the construction and characterization of one of the designed coils. The use of this well known technique in the MRI community allows us to further optimize MPI coils for upscaled MPI scanner, taking various constraint into account. Nevertheless, the properties associated to the stream function used to constrains the numerical optimization are not necessarily representing the final coil properties. Indeed, the discretization of the stream function into the wire centroids and the underlying selection of the fabrication method and material may significantly modify the optimized properties. The whole design optimization of a coil is in fact an iterative process.

From the three designed coils only the cylindrical drive coil fulfill all the required properties of a rabbit MPI scanner. Moreover, the optimized quadrupole will help to alleviate the main challenge to be faced in the upsampling process of FFL scanner: the dissipated power [31], [32].

This method is also a suitable tool to study the field quality needed in order to produce a high quality line for FFL scanners. It will also help to develop shimming coils to quality needed in order to produce a high quality line for scanner: the dissipated power [31], [32].

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